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UNCERTAINTIES IN SATELLITE POSITION DUE TO SOLAR RADIATION
PRESSURE EFFECTS

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-64-147

NOVEMBER 1964

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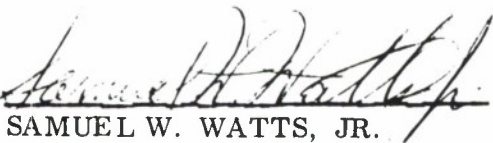
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ABSTRACT

A simple formula for calculating the magnitude of solar radiation pressure is given, and the uncertainty in each of the terms is discussed. Worst-case positional deviations due to solar radiation pressure are calculated for one satellite pass under several sets of conditions. Complicating factors, such as earth shadow, body motion and reflected radiation, are discussed, and finally, the net worst-case uncertainties in predicted satellite position after one pass are estimated.

REVIEW AND APPROVAL

This technical documentary report has been reviewed and is approved.



SAMUEL W. WATTS, JR.

Lt Colonel, USAF

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496L System Program Office

Deputy for Systems Management

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SECTION I

INTRODUCTION

For several reasons, some purely scientific and some military in nature, it would be desirable to be able to predict the position of any close earth satellite to within a few feet with respect to some observer on the earth's surface. To limit the problem somewhat, we shall confine our interest to predictions over a time interval representing only a few passes of the satellite around the earth; i. e. , we shall assume that the satellite has been under observation for a period of time determined primarily by the prevailing situation (for a part of a pass, or, perhaps, for many weeks). On the basis of these observations, we shall attempt to predict the position of the satellite several hours into the future. The physical world, however, has contributed a great many sources of error into our prediction problem, such as errors caused by gravitational anomalies, site-location uncertainties, variations in air drag, charge drag, measurement biases, timing biases, and many others.

This document attempts to examine the positional errors produced by uncertainties in the effects of solar radiation pressure, and, obviously, represents only a small portion of a much larger study program. Because of the short prediction time-span under discussion, resonance effects (involving, for example, both solar pressure and gravitational anomalies), which can produce dramatic changes in satellite lifetime, will not be of any significance in this discussion.

SECTION II

SIMPLE SOLAR RADIATION PRESSURE [1, 2]

INTRODUCTION

The force produced by sunlight falling perpendicularly on a plane surface may be calculated from the following formula:

$$\text{Force, in dynes,} = K \cdot A \cdot \frac{I}{c},$$

where

- K = a scattering constant dependent upon the surface characteristics of the illuminated plane, dimensionless,
- A = area of the illuminated plane surface, cm^2 ,
- I = solar energy flux at the illuminated plane surface, integrated over all frequencies, ergs/cm-sec. , and
- c = velocity of light, cm/sec.

Near the earth, the force exerted by sunlight is about $4.5 \times 10^{-3} \text{ dynes/cm}^2$.

For the case of satellites, a slightly different formula is more convenient:

$$\text{acceleration, cm./sec.}^2 = K \cdot \left(\frac{A}{M} \right) \cdot \frac{I}{c}$$

where

- M = mass of satellite, grams, and
- K = scattering constant which includes the effects of the object's shape.

These formulas are the direct result of that portion of quantum theory which asserts that with each quantum of energy, $E = h\nu$, there is associated a momentum, $h\nu/c$, and the radiation pressure can be computed as the net rate of transfer of momentum through unit area at the point considered. For our purposes we may consider photons to be similar to hard bullets, with ultra-violet light carrying more momentum than infrared light. Since the velocity of light, c , is known to high accuracy, we see that the acceleration produced by simple radiation pressure is influenced by three terms: K , (A/M) , and I . We shall examine these three terms separately.

THE SCATTERING CONSTANT, K

The scattering constant, K , is a function of the surface properties of the satellite. From our view of photons as "hard bullets," we may infer that K can take on values between zero and two, as follows:

If the plane surface upon which the sunlight is falling absorbs all the light perfectly, then $K = 1$. If the surface is perfectly transparent, then the surface absorbs no momentum from the photons, and $K = 0$. (Alternatively, the surface completely absorbs the bullet-like photons and thereby receives momentum, but the opposite side of the plane re-emits an equal number of photons, and the momentum transferred by the emission exactly cancels that gained by the initial collision.) If the plane surface is smooth and shiny and reflects specularly, then $K = 2$ because the surface gains momentum from the initial collision of photons, and gains again when the photons are re-emitted from the same side in the opposite direction.

The case of a spherical satellite is interesting. If the incident light is totally absorbed by the satellite, the satellite experiences a certain pressure, P , in a direction away from the sun. If the satellite specularly reflects the sunlight, it experiences the same pressure P in the same direction as before;

the incident light produces the pressure P , but the reflected light is scattered isotropically away from the sphere, and any momentum transferred in one direction by the reflected light is cancelled by light reflected in the opposite direction.

Figure 1 shows a case in which the net radiation pressure on an opaque object is actually less when the surface is a good reflector than when the surface absorbs the radiation. Surface roughness usually implies a greater measure of diffuse reflection, and the effect on the net radiation pressure experienced by the surface must be examined separately for each case. For example, the force exerted on a shiny surface placed perpendicular to the incident light beam will decrease if the surface is roughened, while roughening the surface of the wedge shown in Fig. 1 would increase the radiation pressure.

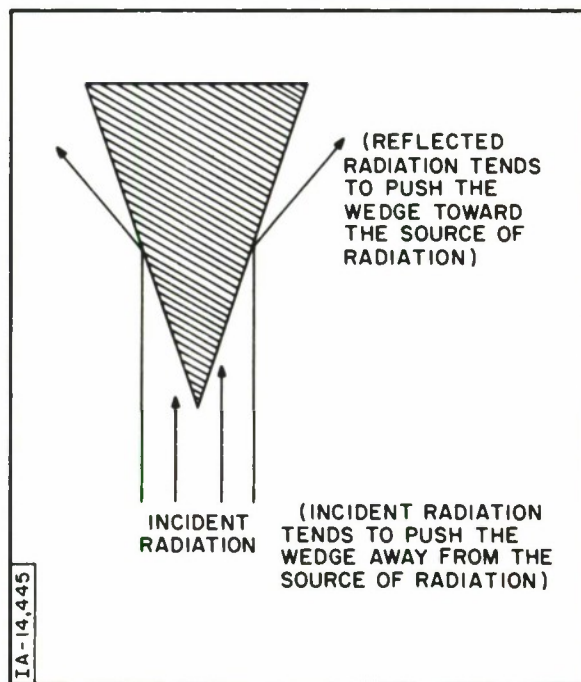


Fig. 1. Case of Reflecting Object Subject to Less Force than Totally Absorbing Object

One could, of course, invent cases in which the surface roughness does not yield isotropic reflection (using some form of regular grating, for instance), and the net effects would be different.

We see from the preceding discussion that, in general, K is dependent upon the shape and surface characteristics of the illuminated body. Unless we have detailed information about the behavior of these variables as a function of time, we would, in principle, be forced to assume that K could take on any value from zero to two. It is difficult, however, to imagine a realistic, practical situation in which K is much less than unity; similarly, K will approach its upper limit only when most of the incident radiation is reflected directly back toward the source, and the more usual situations do not maintain this condition (except, perhaps, in the case of a set of corner reflectors) for any appreciable length of time. Thus, in practice, we might estimate that the average value for K will lie somewhere between 0.5 and 1.5, unless the satellite is specifically designed to thwart such estimates.

It is, perhaps, well to point out that the surface properties of a satellite can change with time. The Echo I balloon, for example, apparently received many puncture wounds from micrometeorites, and gradually changed from a smooth-skinned sphere to a wrinkled bag. While it is not probable that the surface properties will change greatly within several hours, longer term predictions of position could be upset seriously.

THE AREA-TO-MASS RATIO, (A/M)

The area referred to in the expression (A/M) is the cross-sectional area of the illuminated object, i. e., the area which intercepts the radiation. For a non-spherical satellite, the area is a function of the orientation of the body with respect to the illumination source, but in this section we shall discuss only the simple case of a spherical object. Note that we have arbitrarily lumped

"shape" variables – deviations from the simplest case of a plane surface oriented perpendicular to the incident radiation – into the scattering constant K ; since the shape directed toward the radiation is also a function of body orientation, K is not actually a constant. Such factors will be discussed later (see page 15).

It is of interest to examine the range of variation in the ratio (A/M) encountered in practice. Probably thin-skinned balloons and hair-like dipoles represent the largest (A/M) cases of importance. The Echo I balloon, a half-mil Mylar sphere externally coated with an aluminum layer approximately $0.2\text{-}\mu$ thick, initially had a ratio, (A/M) , of $102\text{ cm}^2/\text{gram}$, which increased (as the sublimating powders used to inflate the balloon gradually escaped) to about $125\text{ cm}^2/\text{gram}$.^[3] Dipoles cut from fine wire can apparently be made with (A/M) ratios of several hundred cm^2/gram . It is not known how far the art of making space-balloons can be extended, but, for the present, a practical upper limit to (A/M) of several hundred cm^2/gram will be assumed.

The lower (A/M) limit might be estimated from solid metal spheres. A 25-pound lead sphere, for example, has $(A/M) \sim 0.01\text{ cm}^2/\text{gram}$; a 250-pound sphere has $(A/M) \sim 0.005\text{ cm}^2/\text{gram}$. A more useful estimation of (A/M) values for satellites which are not balloons can be obtained from Table I, which lists the results of crude calculations for satellites selected at random.^[4]

If we were forced to guess the (A/M) ratio for some arbitrary satellite under observation, we would guess the value $(A/M) = 0.1\text{ cm}^2/\text{gram}$ in the absence of any information, based on the following argument: Very few satellites are balloons, and the average dense satellite has about the value given. If it were known that the satellite under observation was Russian, the value chosen might be somewhat lower, perhaps $(A/M) = 0.03\text{ cm}^2/\text{gram}$.

The point of this exercise is that, although the practically obtainable values for (A/M) can vary from about 500 to $0.005 \text{ cm.}^2/\text{gram}$ (a ratio of 100,000:1), the actual variation will be only about 3:1 or 10:1, provided the satellite in question can be correctly designated as balloon or non-balloon. The distinction between balloons (and wire dipoles) and non-balloons is, of course, extremely important from a military point of view, since only the latter can carry weapons. If more information about the satellite is available, then the uncertainty in (A/M) is reduced correspondingly.

Table 1

Approximate Values of (A/M) for a Random Selection of Satellites

Satellite Name	Shape	Dimensions	Weight lb.	Approx. (A/M) Ratio, $\text{cm.}^2/\text{gram}$
Sputnik 1	Sphere	23" diameter	184	0.032
Explorer 1	Cylinder	80" x 6"	30.8	0.22
Sputnik 3	Cone	148" x 68"	2926	0.024
Atlas	Cylinder	80' x 10'	8700	0.19
Vanguard 2	Sphere	29" diameter	21.5	0.21
Discoverer	Cone-Cylinder	19' x 5'	1362	0.14
Lunik 3	Ellipsoid	52" x 47"	614	0.049
Vostok 1	Cone-Cylinder	30' x 10'	10,417	0.059

THE SOLAR ENERGY FLUX, I

The quantity of interest, I, may be defined as the amount of energy received from the sun per cm.²/sec. at the surface of the illuminated satellite. A related quantity called the solar constant, I_0 , is defined as the amount of energy received from the sun per cm.²/sec. just outside the earth's atmosphere at the earth's mean distance from the sun. Clearly, if we assume that the satellite of interest is substantially outside the earth's atmosphere, then

$$I = I(t) = I_0 \left(\frac{R_e}{R(t)} \right)^2$$

where

R_e = the mean distance of the earth from the sun, and

$R(t)$ = the distance of the earth from the sun at time t .

The second-order variations in $R(t)$ caused by the motion of the satellite around the earth may be neglected. The distances R_e and $R(t)$ can be calculated with great accuracy, so that the principal uncertainty in I is a result of the uncertainty in I_0 .

The measurement of I_0 would be relatively simple if it were not for the earth's atmosphere, which absorbs the sun's energy in a frequency-dependent manner. The presence of water vapor in the atmosphere complicates the measurement of the infrared energy flux, and the presence of ozone complicates the measurement of the ultraviolet portion of the spectrum. Several observers have estimated the solar constant, and the value is constantly being refined with results from test-rocket measurements. Some of the values quoted are as follows: [1]

Abbot: 1.938 cal./cm.²/min.

Unsold: 1.900 cal./cm.²/min.,

C. W. Allen: 1.970 cal./cm.²/min.

The value given in the Handbook of Chemistry and Physics^[5] is 2.00 cal./cm.²/min., with a probable uncertainty of ± 2 per cent. Abbott found variations in the solar radiation constant of about ± 2 per cent; there is some evidence that the variation is correlated with the solar cycle, but the matter is still open to doubt.^[1] The solar constant remains "quite constant" (within ± 0.1 per cent, except for short periods of time when fluctuations are as large as a few per cent.^[6] It would appear, from these estimates, that the net uncertainty in I is a few per cent.

SECTION III

EFFECTS OF SIMPLE SOLAR RADIATION PRESSURE ON SATELLITE ORBITS

For what we call the "simple solar radiation pressure case," i. e. , the case of sunlight falling on a spherical satellite in the absence of earth shadow, analytical solutions for the changes in the orbital elements due to radiation pressure have been developed by several authors.^[2] The equations will not be repeated here because we are interested not in the detailed behavior of the satellite's motion, but in semi-qualitative estimates of the gross displacements caused by radiation pressure. A description of the gross changes in a satellite's orbit under the influence of solar radiation pressure is as follows:

In general, during a complete orbital period, solar pressure causes a first-order perturbation of all six orbital parameters. However, the most conspicuous effect for a nearly circular orbit is a displacement of its geometric center. This displacement is perpendicular to the earth-sun line in the orbit plane and in a direction such as to decrease the altitude of that part of the orbit in which the satellite moves away from the sun. The mean radius is almost unaffected for orbits of small eccentricity.^[7]

Figure 2 illustrates the path of a satellite acted upon by radiation pressure.^[8]

In another report,^[9] the following statement is made:

In general the sun does not lie in the nominal orbit plane. If the acceleration due to radiation pressure is resolved into components lying in the orbit plane and a component normal to the orbit plane, the in-plane acceleration component will be smaller than the magnitude previously considered, and consequently the perturbations will be decreased. In RAND Research Memorandum RM-2439, The Effects of Radiation Pressure on Earth Satellites (u), the normal component was found to be relatively

unimportant, causing only very small periodic fluctuations of the node and inclination angle. The principal effect of three-dimensional considerations is to diminish the magnitude of the in-plane perturbations by an amount corresponding to the misalignment of the orbit plane with respect to the sun.

For nearly circular orbits, the magnitude* of the velocity of the geometrical center of the orbit (moving perpendicular to the earth-sun line, as shown in Fig. 2) is given by [7]

$$|\dot{c}(\theta)| = P \cdot \left(\frac{A}{M} \right) \cdot \frac{3}{2n} \cdot \cos \theta \text{ m./sec.},$$

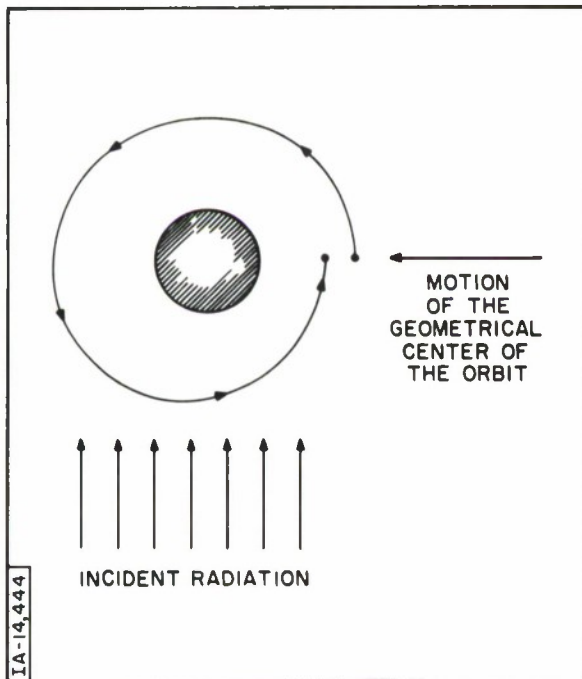


Fig. 2. Geometrical Relationship Between Incident Radiation and the Motion of the Satellite Orbit

*For a spherical satellite with $K = 1$, and no earth shadow.

where

- P = the solar radiation pressure near the earth
 $(P \sim 4.5 \times 10^{-5} \text{ dyne/cm.}^2,$
- θ = the angle between the earth-sun line and the
 orbit plane, and
- n = the mean motion of the satellite:

$$n = \sqrt{\mu a}^{-3/2} = \frac{2\pi}{T} \text{ radians/sec. ,}$$

where

- μ = $3.98946 \times 10^{14} \text{ cu. m. /sec.}^2,$
- a = the semi-major axis of the orbit, m. , and
- T = the time required for one satellite pass, (sec.).

From an inspection of Fig. 2, it is apparent that the maximum physical displacement of the satellite's position from its unperturbed orbit during one pass will be roughly equal to the displacement of the center of the orbit during the time required for one pass. Thus, in the shadowless case, where the sun is in the satellite's orbit plane (a worst-case situation), the maximum displacement, D, of the satellite because of solar radiation pressure during one pass will be given by

$$D = T \left| \dot{c} \right| = P \cdot \left(\frac{A}{M} \right) \cdot \frac{3T}{2n} = \frac{3\pi}{\mu} \cdot P \cdot \left(\frac{A}{M} \right) \cdot a^3 \text{ meters.}$$

The formula for D was calculated for several values of (A/M) and a , and the results are shown in Table 2. The Table represents worst-case errors in position (in meters) after one pass, for various orbit altitudes (in statute miles above the earth's surface). The three values used for (A/M) represent (a) the largest practical value likely to be used, (b) the value for Echo I, and (c) a pessimistically large value of (A/M) for non-balloon satellites.

Table 2
Worst-Case Positional Deviations after One Satellite Pass

(A/M) , cm. ² /gram	Height above Earth's Surface:			
	100 miles	300 miles	Echo I (903) miles	3000 miles
300	893	1030	1531	4493
Echo I (125)	372	429	638	1872
0.5	1.5	1.7	2.6	7.5

Several conclusions may be drawn at once on the basis of Table 2. First, solar radiation pressure acting upon non-balloon satellites causes virtually negligible positional errors after one pass, regardless of altitude; even the crudest compensation for radiation pressure would suffice to reduce the positional errors to fractions of one meter after one pass. Second, quite appreciable errors in position are found after one pass for balloon-type satellites, regardless of altitude; adequate prediction of their positions will require rather careful and sophisticated compensation for solar radiation pressure.

SECTION IV

COMPLICATIONS ADDED TO THE SIMPLE RADIATION-PRESSURE CASE

The computations made in the previous section were based on the simplest possible model: constant force in a direction parallel to the incident direct radiation. In actual practice, several severe deviations from this model are encountered. Their effects and the manner in which they may be handled are discussed below.

VARIATION IN EFFECTIVE CROSS SECTION

As discussed previously, the reflection coefficient K includes contributions from the surface properties of the satellites, as well as the shape of the object and its aspect with respect to the sun. Consequently, the magnitude of the force (proportional to K) exerted by radiation pressure cannot, in general, be estimated accurately unless detailed knowledge about the satellite's surface, size, shape and body motion is available. If such detailed information were available, computer programs using numerical evaluation techniques could compensate for variations in effective cross section exceedingly well.

OBLIQUE FORCES

Except for objects whose shape possesses certain symmetry properties, the force exerted by solar radiation pressure is not parallel to the sun's incident rays, i. e. , K may be thought of as a time-varying tensor. While computer programs may readily be developed to calculate the effects of oblique forces, it is evident that here also we must have detailed knowledge of the satellite's surface properties, size, shape and body motion.

SHADOW

When the satellite's position falls within the shadow cast by the earth, the direct solar radiation pressure vanishes, thus providing another deviation from the "constant force" model. Fortunately, the variables in this portion of the problem are simply the relative positions of the sun, the earth and the satellite, all of which are easily calculable with great precision.* The exact calculations are extremely awkward to program on a computer in an efficient and economical manner, so most programmers resort to approximations (such as representing the shadow region as a truncated cylinder rather than as a truncated cone) which are entirely adequate for general use. The result is that, although serious positional errors could accumulate if the effects of shadow were ignored, compensations for shadow regions can be so exact as to virtually eliminate any positional uncertainties from this source. [2]

It might be mentioned that the moon and other astronomical bodies can also cast shadows, and that a truly general-purpose program would include their effects as well.

REFLECTED RADIATION

When the sun's rays strike the earth, they are partly absorbed and partly reflected, and the reflected portion is scattered diffusely from rough land masses or clouds and specularly from smooth water. Many more measurements, presumably made from satellites or rocket probes, will be necessary before reasonably quantitative estimates for reflected radiation can be given. It would appear that the net magnitude of reflected radiation would vary with cloud cover,

*Effects due to refraction by the atmosphere, diffraction by the earth and the oblateness of the earth may be included if desired.

the weather-dependent surface properties of large bodies of water, the seasonal coloring of vegetation, and the particular aspect of the earth facing the sun at a given moment. Furthermore, the reflected sun's rays must pass twice through the earth's atmosphere and thereby suffer considerable angle-dependent modification.

Workers at Lincoln Laboratory^[2] ran many trials with a computer program designed to model at least the gross effects of reflected radiation, using values for the parameters of the problem which were thought to encompass the reasonable upper and lower limits. They found that reflected radiation rarely contributed changes to the orbital elements larger than 1 per cent of the changes caused by direct sunlight, but for some low-altitude satellites, the effects amounted to as much as 5 per cent. The uncertainty in the magnitude of the reflected-radiation effects was estimated by Dr. Jones to be about equal to the effects themselves.

ISOTROPIC EARTH RADIATION

The vast energy intercepted from the sun's rays by the earth goes, for the most part, toward heating the earth, and a large fraction of this energy is reradiated (in the form of infrared radiation) in a roughly isotropic fashion. If the reradiation were truly isotropic and constant, and if the effective (A/M) ratio of the satellite as seen from the earth were constant, then the radiation pressure contributed by the hot earth would be entirely equivalent to a negligible decrease in the earth's mass.

Preliminary investigations of the infrared radiation from the earth (Intermountain Weather, Inc. , Final Report. . .) indicate that at 1000 km this radiation is within ± 10 per cent of the uniformity, [3]

Since the reradiated power is only a small fraction of that of direct radiation, the effect of the former on satellite orbits will undoubtedly be small, but may possibly have to be taken into account for precision work.

MASKING EFFECTS

One might dismiss many of the previously discussed uncertainties – or, at least, reduce the variability of many parameters – by attempting to measure the solar radiation effects during the observation period prior to the prediction interval. Certainly, some valid estimates can be made if the observation period is long enough (some of the references include observational estimates), but anything better than a rough guess will be difficult to achieve because of the masking effects of air drag, gravitational anomalies and other perturbing forces. It is impossible to say, at present, just how useful pre-predication measurements would be in estimating solar radiation pressure effects (except, perhaps, to distinguish between balloons and non-balloons), but considerable refining of the whole field of satellite orbital mechanics will be necessary before measurements taken over a few passes can yield estimates of future position which are accurate to a few per cent.

SECTION V

RESULTANT UNCERTAINTY IN SATELLITE POSITION

An attempt will be made, in this section, to estimate the net uncertainty in position which results from the uncertainty in many of the variables discussed in previous sections. The positional errors will depend upon the extent of our knowledge of the target parameters in a particular situation, hence, three different cases of interest are discussed. It is to be emphasized that the numerical estimates refer to worst-case or almost worst-case situations, with the sun in the orbit plane and the earth's shadow omitted. Since shadow effects introduce no appreciable uncertainty in position, the addition of the earth's shadow would tend to decrease the positional uncertainty because of other variables. If the sun is not in the orbital plane, the positional uncertainties will be reduced by a factor which is roughly equal to $(\cos \theta)$, where θ is the angle between the satellite orbit plane and the earth-sun line. [7]

CASE I: TARGET CHARACTERISTICS AND BODY MOTION COMPLETELY KNOWN

We assume that K and the direction of the force caused by radiation pressure are known exactly as functions of time, despite body motion. With these variables conveniently eliminated, we are left with the following uncertainty budget:

	<u>High Orbits, Per Cent</u>	<u>Low Orbits, Per Cent</u>
Uncertainty in I:	± 2	± 2
Uncertainty in Reflected Radiation:	± 1	± 5
	<hr/> ± 3	<hr/> ± 7

Table 3 shows the resultant positional uncertainties, where "high orbit" is taken as 3000 miles and "low orbit" is taken as 100 miles to coincide with Table 2.

Table 3

Maximum Positional Uncertainties After One Pass for Case I.

(A/M), cm. ² /gram	Low Orbit, miles	High Orbit, miles
300 cm. ² /gram	± 63.0	± 135.0
0.5 cm. ² /gram	± 0.1	± 0.2

As mentioned previously, $(A/M) = 300 \text{ cm.}^2/\text{gram}$ is about the largest practical value likely to be encountered at present (Echo I has $(A/M) \sim 125 \text{ cm.}^2/\text{gram}$), while $(A/M) = 0.5 \text{ cm.}^2/\text{gram}$ represents a pessimistically large value for a non-balloon target. Thus, here too, we have taken worst-case values.

CASE II: TARGET CHARACTERISTICS COMPLETELY KNOWN, BODY MOTION UNKNOWN

This situation corresponds to the case of a friendly test vehicle (such as a rocket body launched by the United States) whose orientation as a function of time is not known. If the target is spherical with a uniform surface, then Case II is equivalent to Case I. Our knowledge of the body size, shape and surface permits us to calculate the average value of K and (A/M) ; the errors then result from the fact that a time period of two or three passes may not be sufficient to yield average results, especially if the target is partially stabilized. For a specific target, a probabilistic measure of the error committed by using average values for K and (A/M) could be calculated using a Monte Carlo method. Such calculations would, undoubtedly, demonstrate large errors for some special situations of initial target orientation and body motion. It seems reasonable, however, to

assume that for most targets and for most passes the value for $K(A/M)$, averaged over only a few passes, will not be more than a factor of three larger or smaller than the value obtained by truly averaging over all situations.

Since our previous calculations have been based on worst-case models, the worst-case positional uncertainty for Case II is identical to that given in Table 2. For a specific situation in which the total displacement, D , in some plane because of solar radiation pressure is small compared to the maximum values shown in Table 2 (caused by shadow and a non-favorable geometry), then the uncertainty could be estimated as about $\pm 3D$.

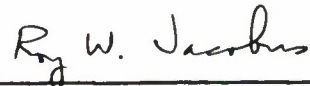
CASE III: NOTHING KNOWN A Priori ABOUT THE TARGET

We are forced, in this case, to make estimates about all characteristics of the target which might affect solar radiation pressure, except the orbital parameters. Again the worst-case positional uncertainties are those given in Table 2; i. e., the uncertainty corresponds to ignoring solar radiation pressure altogether. As discussed in a previous section, however, we may estimate an average $(A/M) \sim 0.1 \text{ cm.}^2/\text{gram}$ with the assurance that, in most cases (non-balloons), the correct value will differ by a factor of, at most, ten. Thus, the total uncertainty in $K(A/M)$ is roughly 30:1, the result of a 10:1 uncertainty in the average value (for non-balloons) and a 3:1 uncertainty about deviations from the average. For a specific displacement, D , caused by solar radiation pressure, the uncertainty could be estimated as about $\pm 30D$.

SECTION VI

CONCLUSIONS

As shown in Table 2, the total effects of solar radiation pressure after several passes may be ignored entirely for any satellite whose (A/M) ratio is $0.5 \text{ cm.}^2/\text{gram}$ or less, a condition which appears to be met by virtually all non-balloon targets. Balloons with a large (A/M) ratio experience positional perturbations of as much as several miles during one pass, and solar radiation pressure certainly cannot be neglected in such cases. Very sophisticated corrections may be carried out to account for various components of the sunlight perturbations, but unless the object is spherical or unless its characteristics and body motion are well known, corrections accurate to better than an order of magnitude will be most difficult to obtain. Consequently, the accuracy of predicted positions of irregularly shaped balloons at high altitude may be seriously limited by the effects of solar radiation pressure, unless better means for estimating some of the variables can be found.



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